PB82-207770

On the Proportionality of Fine Mass Concentration and Extinction Coefficient for Bimodal Size Distributions

(U.S.) Environmental Sciences Research Lab.
Research Triangle Park, NC

1981

U.S. Department of Commerce
National Technical Information Service
NTIS

TECHNICAL REPORT DATA
(Please read Instructions on the reverse before completing)

1. REPORT NO. EPA-600/J-81-125	2. Journal Article
	PB82-207770
4. TITLE AND SUBTITLE ON THE PROPORTIONALITY OF FINE MASS CONCENTRATION AND EXTINCTION COEFFICIENT FOR BIMODAL SIZE DISTRIBUTIONS	5. REPORT DATE April 1982
	6. PERFORMING ORGANIZATION CODE
7. AUTHOR(S) Charles W. Lewis	8. PERFORMING ORGANIZATION REPORT NO.
9. PERFORMING ORGANIZATION NAME AND ADDRESS Environmental Sciences Research Laboratory--RTP,NC Office of Research and Development U.S. Environmental Protection Agency Research Triangle Park, N. C. 27711	10. PROGRAM ELEMENT NO. CDTA1D/09-1168 (FY-81)
	11. CONTRACT/GRANT NO.
12. SPONSORING AGENCY NAME AND ADDRESS Environmental Sciences Research Laboratory -- RTP,NC Office of Research and Development U.S. Environmental Protection Agency Research Triangle Park, N. C. 27711	13. TYPE OF REPORT AND PERIOD COVERED
	14. SPONSORING AGENCY CODE EPA - 600/09

15. SUPPLEMENTARY NOTES
REFERENCE: Atmospheric Environment, 15(2):2639-2646, 1981.

16. ABSTRACT

For a bimodal size distribution of ambient aerosol an upper limit in particle size can be chosen for the fine aerosol fraction so that the extinction coefficient for light scattering and absorption is directly proportional to the fine mass concentration, with no dependence on the coarse mass concentration. With the size parameters of the modes taken to be "grand average continental" values of Whitby, the upper size limit is 3.4 um (aerodynamic diameter). Consequently, the extinction coefficient could be inferred from a measurement of the fine mass concentration only, even in arid regions where coarse aerosol concentrations usually far exceed the fine.

17. KEY WORDS AND DOCUMENT ANALYSIS

a. DESCRIPTORS	b. IDENTIFIERS/OPEN ENDED TERMS	c. COSATI Field Group

REPRODUCED BY
NATIONAL TECHNICAL
INFORMATION SERVICE
U.S. DEPARTMENT OF COMMERCE
SPRINGFIELD, VA 22161

18. DISTRIBUTION STATEMENT RELEASE TO PUBLIC	19. SECURITY CLASS (This Report) UNCLASSIFIED	21. NO. OF PAGES 10
	20. SECURITY CLASS (This page) UNCLASSIFIED	22. PRICE

EPA Form 2220-1 (9-73)

Atmospheric Environment Vol. 15, No. 12, pp. 2639–2646, 1981
Printed in Great Britain.

0004-6981/81/122639-08 $02.00/0
Pergamon Press Ltd.

EPA-600/J-81-125
Journal Article

ON THE PROPORTIONALITY OF FINE MASS CONCENTRATION AND EXTINCTION COEFFICIENT FOR BIMODAL SIZE DISTRIBUTIONS

C. W. LEWIS

Environmental Sciences Research Laboratory, U.S. Environmental Protection Agency, Research Triangle Park, NC 27711, U.S.A.

(*First received* 13 *March* 1981 *and in final form* 5 *June* 1981)

Abstract—For a bimodal size distribution of ambient aerosol the upper particle size limit of the fine aerosol fraction can be chosen in such a way that the extinction coefficient for light scattering and absorption is directly proportional to the fine mass concentration, regardless of the magnitude of the coarse mass concentration. With the size parameters of the modes taken to be the "grand average continental" values of Whitby, the upper size limit is 3.4 μm (aerodynamic diameter). Consequently, the extinction coefficient could be inferred from a measurement of the fine mass concentration only, even in arid regions where coarse aerosol concentrations usually far exceed the fine.

INTRODUCTION

It is now generally recognized that light scattering by the ambient atmosphere is primarily due to particles with diameters in the size decade 0.1–1 μm. The dominance of this size range occurs both because the scattering cross-section per aerosol particle volume attains a sharp maximum in this range for visible light (Willeke and Brockmann, 1977), and because a significant fraction of the ambient aerosol mass (volume) is usually present in the same range. This circumstance has led investigators to examine experimentally the possibility that the fine particle aerosol mass concentration may be linearly related to the amount of light scattering, as indicated by the scattering extinction coefficient for particles, b_{sp}. (Waggoner and Weiss, 1980; Dzubay and Clubb, 1980).

Aerosol particles can also interact with light through absorption. Like scattering, this mode of interaction is more effective for fine than coarse particles, although the effect of absorption is usually smaller than that of scattering. These considerations naturally lead to the possibility of a direct proportionality between fine mass concentration and the *total* extinction coefficient for particles, b_{ep}, which includes the effects of absorption as well as scattering,

$$b_{ep} = b_{sp} + b_{ap}. \quad (1)$$

The advantage of dealing with the total extinction coefficient is that it is closely related to one of the most common measures of visual air quality, the visual range V_R. The relation is given by the Koschmieder equation

$$V_R = 3.91/b_e, \quad (2)$$

where b_e differs from b_{ep} only by an additional small term accounting for the influence of air molecules (principally Rayleigh scattering).

Much of the experimental support for an extinction-fine mass relationship comes from measurements performed in regions where the coarse mass concentration is typically less than or comparable to the fine mass concentration. Consequently, it is not certain how applicable the relationship may be for arid regions with small anthropogenic impact, since the coarse mass concentration may be much greater than the fine in such areas. It is precisely these arid areas, exemplified by parts of the western U.S.A. containing spectacular natural vistas, that are the focus of recent national legislation to protect visibility (Frederick, 1979).

The following analysis shows that the upper particle size limit of the fine aerosol fraction can be chosen in such a way that the extinction coefficient is strictly proportional to the fine mass concentration, regardless of the magnitude of the coarse mass concentration. This surprising result increases the attractiveness of fine mass concentration as a surrogate of visual air quality. Although the analysis leads to what may be regarded as an optimum choice for the fine-coarse cutpoint of a mass sampler (considering only visibility needs), other considerations may argue against the choice in some circumstances. Thus, out of convenience one may prefer to use an existing mass sampler with a cutpoint which only approximates the theoretically optimum one. Also, since the optimum cutpoint does not quite coincide with the usual bimodal minimum the usefulness of the resulting mass samples for purposes other than visibility monitoring (e.g. source identification) may be somewhat compromised. For such reasons the effect of a cutpoint different from the optimum value is also investigated.

THEORETICAL

The particle extinction coefficient is conventionally calculated from the expression

$$b_{ep} = \int_0^\infty \sigma_e (dN/dD)\,dD, \tag{3}$$

where dN/dD is the particle number concentration distribution and σ_e is the single particle extinction cross section, both being functions of particle diameter D. Following Willeke and Brockmann (1977), it is convenient to rewrite Equation (3) in a volumetric representation and to indicate separately the contributions to b_{ep} made by the accumulation and coarse modes of the usual bimodal distribution of ambient aerosol particle sizes. This gives

$$b_{ep} = \sum_{j=1}^{j=2} (M_j/\rho_j) I_j, \tag{4}$$

where $j = 1$ for accumulation mode, $j = 2$ for coarse mode, $M_j\,[\mathrm{g\,m^{-3}}]$ = total mass concentration in the jth mode, $\rho_j\,[\mathrm{g\,cm^{-3}}]$ = average density of a particle in jth mode.

$$I_j[\mu\mathrm{m}^{-1}] = \int_0^\infty (\sigma_e/V)(dV_j/dD)\,dD \tag{5}$$

$\sigma_e\,[\mu\mathrm{m}^2]$ = extinction cross section for single particle with diameter D and refractive index $n - ki$

$$V[\mu\mathrm{m}^3] = \pi D^3/6 \tag{6}$$

$dV_j/dD\,[\mu\mathrm{m}^{-1}]$ = normalized volume distribution of particles in jth mode, with

$$\int_0^\infty (dV_j/dD)\,dD = 1. \tag{7}$$

With the quantities dimensioned as indicated, b_{ep} has units of m^{-1}.

1. *Derivation of optimum cutpoint*

The basic result that the extinction coefficient can be made independent of coarse mass concentration follows directly from (4) by a simple qualitative argument. It is apparent from (4) that the relative contribution to extinction from mass in the coarse mode, compared with mass in the accumulation mode, is given by the ratio $I_2/\rho_2 : I_1/\rho_1$, whose numerical value will be seen to be of order 10^{-1}. This fraction depends on the relative size distribution of mass within each mode, but not on the relative nor absolute total amounts of mass in the two modes. Therefore, if only this fraction of the coarse mode mass is taken (by adjusting the sampler fine-coarse cutpoint so that just this fraction of the entire coarse mode relative size distribution is included in the fine mass sample), the mass makes the same contribution to extinction as if it were in the accumulation mode. The remaining coarse mode mass above the cutpoint may then be ignored, because its effect has already been taken into account. The cutpoint also causes the mass sampled from the accumulation mode to be attenuated, but this involves only a small correction which can be easily handled. Thus, the choice of cutpoint is determined by the relative shapes and positions of the two modes, but is independent of the mass concentrations associated with them.

To obtain quantitative results a more mathematical formulation is needed. Consider an aerosol sampler which is able to measure both the mass concentration M_f of all particles with diameters between 0 and D_0 and the mass concentration M_c of all particles between D_0 and D_c. An obvious example is the commercial dichotomous sampler for which values of $D_0 = 2.5\,\mu\mathrm{m}$ and $D_c = 15\,\mu\mathrm{m}$ have been chosen so that M_f and M_c approximate M_1 and M_2, respectively. The distinction between the *measured* values M_f and M_c, and the corresponding *modal* values M_1 and M_2, needs to be remembered to avoid confusion in the following. In any case, M_f and M_c can always be written as

$$M_f = M_1 i_1 + M_2 i_2 \tag{8a}$$

$$M_c = M_1 i_1' + M_2 i_2' \tag{8b}$$

where

$$i_1 = \int_0^{D_0} (dV_1/dD)\,dD \tag{9a}$$

$$i_2 = \int_0^{D_0} (dV_2/dD)\,dD \tag{9b}$$

$$i_1' = \int_{D_0}^{D_c} (dV_1/dD)\,dD \tag{9c}$$

$$i_2' = \int_{D_0}^{D_c} (dV_2/dD)\,dD. \tag{9d}$$

The primed and unprimed notation used with the i's indicates which one of two sets of integration limits are used, while the subscript identifies the aerosol mode over which the integration is performed. For the moment, we ignore the complication that most fractionating aerosol samplers operate on the basis of *aerodynamic* diameter rather than *geometrical* (or optical) diameter which is assumed in all the preceding equations.

Solving (8a) and (8b) for M_1 and M_2 and substituting in (4), we have

$$b_{ep} = \frac{1}{\Delta}(I_1 i_2'/\rho_1 - I_2 i_1'/\rho_2) M_f + \frac{1}{\Delta}(I_2 i_1/\rho_2 - I_1 i_2/\rho_1) M_c \tag{10}$$

where $\Delta = i_1 i_2' - i_2 i_1'$.

Aside from the initial hypothesis that the ambient aerosol has a bimodal size distribution, (10) is completely general and can be used to calculate b_{ep} for arbitrary choices of D_0 and D_c. The most interesting feature of (10), however, is the structure of the coefficient of M_c. When reasonable choices are made for particle density, refractive index and relative size distribution of the two modes, it is always possible to select a particular value for D_0 such that the coefficient is zero. In this case the extinction coefficient may be found from a measurement of M_f only, no matter how large M_c happens to be, in agreement with the qualitative argument given previously.

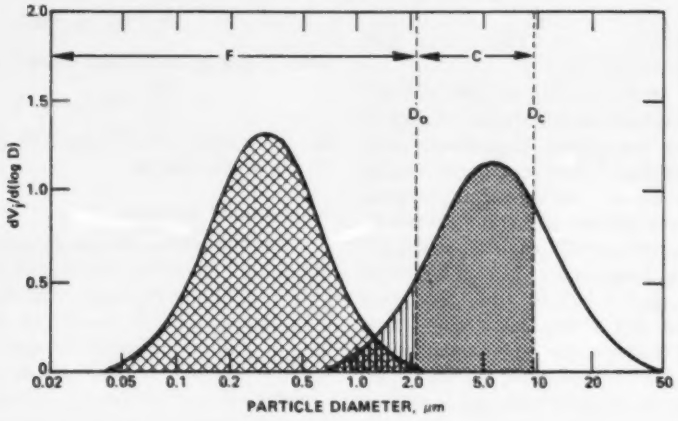

Fig. 1. Plots of Equation (12) for accumulation and coarse mode parameters from Table 1. The integrals i_1, i_2, and i_2' are indicated by cross-hatching, vertical-hatching, and dot-shading, respectively. D_0 and D_c are geometrical diameter cutpoints equal to 2.12 ($3.44/\sqrt{2.64}$) and 9.23 ($15/\sqrt{2.64}$) μm, respectively.

The analysis can be continued by modeling the relative size distributions with log-normal functions. With the replacement

$$\frac{dV_j}{dD}dD = \frac{dV_j}{d(\log D)}d(\log D), \tag{11}$$

we assume the volume size distributions can be written in the normalized form

$$\frac{dV_j}{d(\log D)} = \frac{1}{\sqrt{2\pi}\log\sigma_j} \times \exp\left[-(\log D/DGV_j)^2/2\log^2\sigma_j\right]. \tag{12}$$

Based on hundreds of size distribution measurements at many urban and rural sites in the U.S.A., Whitby (1978) has given a set of "grand average continental" values for the parameters DGV and σ occurring in (12) for both the accumulation and coarse particle modes of ambient aerosol. The values are reproduced in Table 1, and have been used in plotting (12) for both the accumulation and coarse modes as shown in Fig. 1. The figure also shows in a graphical manner the integrals defined by (9a)–(9d).

Table 1 gives in addition assumed values for the remaining parameters required in the calculation, namely, the average density and refractive index for each mode. Since ammonium sulfate and silicon dioxide (quartz) are often the dominant compounds in the fine and coarse modes, respectively (Lewis and Macias, 1980), the density for each mode has arbitrarily been set equal to the bulk density of the corresponding compound. These choices are generally consistent with the commonly assumed value of $2\,g\,cm^{-3}$ for ambient aerosol as a whole, without size differentiation. Following Willeke and Brockmann (1977) and others, a complex refractive index of 1.5–0.02i is assumed, the same for both modes. (Refractive indices for bulk ammonium sulfate and quartz are 1.52 and 1.54, respectively.)

Table 1. Assumed values of ambient aerosol physical parameters

Parameter	Mode Accumulation (1)	Coarse (2)
DGV, μm	0.31*	5.7*
Volume weighted median diameter		
σ	2.0*	2.2*
Geometrical standard deviation		
ρ, $g\,cm^{-3}$	1.77†	2.64‡
Aerosol mass density		
$n-ki$	1.5–0.02i	1.5–0.02i
Refractive index		
I, μm^{-1}	5.5	0.88
Equation (5)		

* Whitby (1978).
† Density of $(NH_4)_2SO_4$.
‡ Density of SiO_2 (nat. quartz).

In (10), b_{ep} will be independent of M_c if

$$\frac{\rho_1}{\rho_2}\frac{I_2}{I_1}i_1 = i_2. \tag{13}$$

The integrals I_1 and I_2 can be immediately evaluated using graphs given by Willeke and Brockmann (1977), which have been developed for log-normal size distributions and a wide range of refractive indices. The graphical evaluation requires a choice of wavelength to be made which, as usual, is taken to be $\lambda = 0.55$ μm, the wavelength of maximum response of the human eye. (Even though I_1 and I_2 occur as a ratio in (13) their wavelength dependence does not cancel.) The resulting values for I_1 and I_2 are given in Table 1, from which we have

$$\left[\frac{\rho_1}{\rho_2}\frac{I_2}{I_1}\right]_{\lambda=0.55\,\mu m} = 0.107. \quad (14)$$

The value of D_0, for which (13) is satisfied, may now be found by plotting each side of (13) and locating the intersection of the two graphs. Logarithmic probability paper is particularly convenient for this purpose since the normalized cumulative log-normal functions i_1 and i_2 will give straight lines such that $i = 0.159, 0.5$ and 0.841 at $D_0 = DGV/\sigma, DGV$ and $DGV.\sigma$, respectively. The result is shown in Fig. 2, from which the critical value $D_0(\text{geo}) = 2.12\,\mu m$ (the *geometrical* diameter for which (13) is satisfied), is immediately obtained. Figure 2 illustrates that i_1 is essentially equal to unity in the region of intersection, causing the left hand side of (13) to be virtually independent of D_0 in the region of interest.

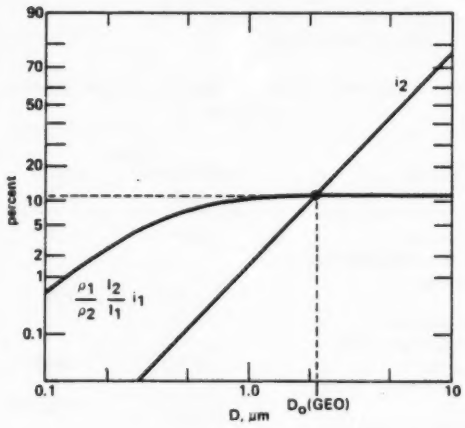

Fig. 2. Graphical solution of Equation (13) for D_0 (geo) using logarithmic probability paper.

Finally, the last step is to transform the critical geometrical diameter to an aerodynamic diameter to be consistent with the type of size classification used in most size fractionating aerosol mass samplers. Ignoring the relatively unimportant effect of the slip correction factor, we have

$$D_0(\text{aero}) = D_0(\text{geo})\sqrt{\rho_2'} \quad (15)$$

from which

$$D_0(\text{aero}) = 3.44\,\mu m. \quad (16)$$

A consideration of the relative sizes of the parameters entering the coefficient of M_f in (10) gives a further simplification of that equation. With D_0 chosen as just indicated, $I_2 i_1'/\rho_2$ is less than 1% of $I_1 i_2'/\rho_1$, and $i_2 i_1'$ is less than 1% of $i_1 i_2'$. The exact values of i_1' and i_2' depend, of course, on the value of D_c, the coarse particle cutoff for the sampler, but the previous statement is insensitive to the particular choice for D_c. Thus, (10) reduces to the very simple form

$$b_{ep} = (I_1/\rho_1)M_f. \quad (17)$$

With I_1 and ρ_1 taken from Table 1 we have

$$\frac{M_f}{b_{ep}} = 0.32\,\text{g m}^{-2}. \quad (18)$$

Equations (15), (16), (17) and (18) are the principal results of this analysis.

2. *Alternative cutpoints*

If the fine-coarse cutpoint D_0(aero) is chosen different from the value given by (15) and (16), then of course b_{ep} will *not* be independent of the coarse mass concentration. In that case (8), (9) and (10) must be used to calculate the impact of coarse mass on b_{ep}. For example, in the important case of the dichotomous sampler (D_0(aero) = 2.5 μm, D_c(aero) = 15 μm) the latter equations together with the parameter values from Table 1 lead to

$$b_{ep}[1/\text{m}] = 3.12\,M_f + 0.265\,M_c \quad (19)$$

in which M_f and M_c are expressed in g m^{-3}. Although (19) allows the use of an existing sampler for extinction monitoring, the disadvantage is that two mass measurements are needed, with the accuracy of one of them (M_c) likely to be affected by the problems of obtaining unbiased samples of large particles under ambient conditions (Wedding *et al.*, 1980).

Figures 3(a) and 3(b) give an alternative view of judging the effect of a non-optimum fine-coarse cutpoint. For selected values of D_0(aero) the quantity M_f/b_{ep} is plotted in both figures, first as a function of the *measured* mass ratio M_c/M_f in Fig. 3(a), and then as a function of the *modal* mass ratio M_2/M_1 in Fig. 3(b). Figure 3(a) emphasizes that M_c/M_f is constrained by (8) to a finite range even though M_2/M_1 in principle can vary from zero to infinitely large. As an example in the use of Figs 3(a) and 3(b), consider once again a 2.5 μm cutpoint. If b_{ep} were inferred from a measurement of M_f (with M_c unmeasured), the true value of b_{ep} would be increasingly underestimated as M_c/M_f increases, reaching a 10% underestimate when M_c is only 30% greater than M_f, or equivalently when M_2 is twice M_1. The use of a 1 or 5 μm cutpoint, however, is seen to result in even larger errors.

EXPERIMENTAL COMPARISON

Waggoner and Weiss (1980) recently reported an approximate proportionality between b_{sp} measured with a heated integrating nephelometer, and M_f measured with a dichotomous sampler whose coarse-fine cutpoint was 2.5 μm aerodynamic diameter. The experimental result, based on many measurements made in different parts of the U.S.A., is given by

$$\frac{M_f}{b_{sp}}(\text{exp}) = 0.36 \pm 0.02\,\text{g m}^{-2}. \quad (20)$$

The proportionality constant in (20) is larger by a factor of 2.2/1.95 than the reported result because a

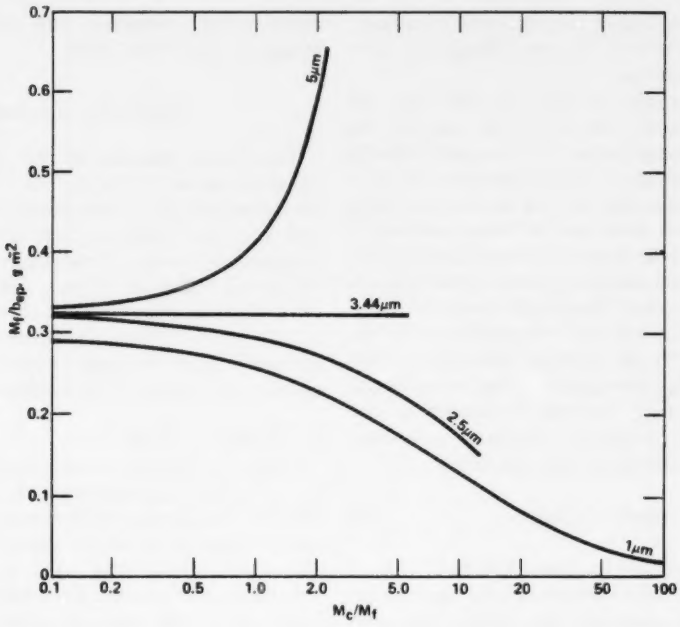

Fig. 3a. Dependence of M_f/b_{ep} on measured mass ratio M_c/M_f for selected values of D_0 (aero). Calculation are based on parameters from Table 1 and D_c (aero) = 15 μm.

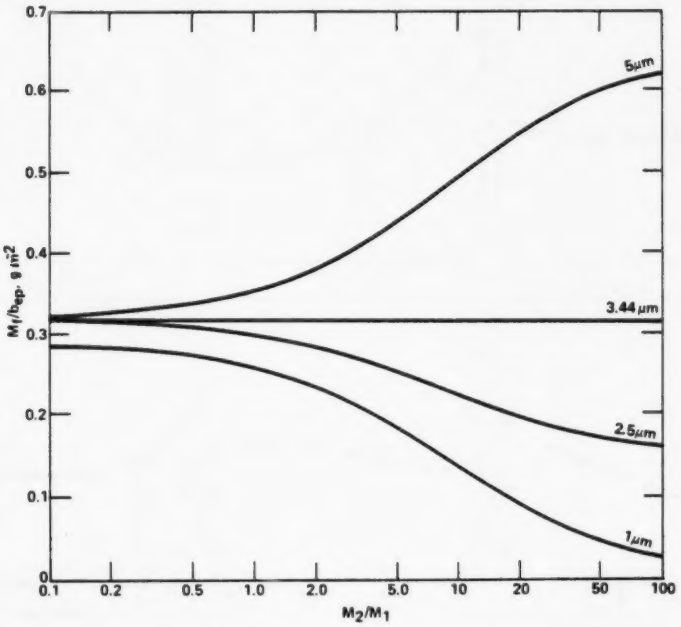

Fig. 3b. Dependence of M_f/b_{ep} on modal mass ratio M_2/M_1 for selected values of D_0 (aero). Calculations are based on parameters from Table 1 and D_c (aero) = 15 μm.

temperature correction for the nephelometer measurements was inadvertently omitted (Waggoner, 1980 private communication).

The proportionality constants in (18) and (20) are not immediately comparable because (1) the dichotomous sampler cutpoint differs slightly from the optimum value of (16), (2) the measurement of b_{sp} by a nephelometer is systematically low because of internal angular truncation error and inefficient sampling of large particles, (3) the nephelometer wavelength is 0.53 rather than 0.55 μm and (4) b_{ep} occurs in one equation, while b_{sp} is in the other. The last difference is probably the most important and can be reconciled as follows. Brockmann (1976) has provided information from which not only b_{ep}, but b_{sp} as well, may be calculated. Using Brockmann's graphs and the parameters from Table 1, one can proceed in a manner entirely analogous to that which led to (18), and obtain

$$\frac{M_f}{b_{sp}}(\text{theo}) = 0.36\,\text{g m}^{-2}, \tag{21}$$

which agrees with the experimental result of (20).

Dzubay and Clubb (1980) have also reported integrating nephelometer-fine mass results from summertime measurements obtained at Research Triangle Park, North Carolina. Their data indicated a M_f/b_{sp} value of 0.23 g m^{-2}. The smaller value may be related to less heating of the aerosol in the integrating nephelometer, compared with the conditions of Waggoner and Weiss (1980).

SENSITIVITY ANALYSIS

The results obtained in (16) for the value of D_0 (aero), and in (18) for the ratio M_f/b_{ep}, depend of course on the values of the parameters given in Table 1. Although these values are considered to be the best experimental average characteristics of ambient aerosol, actual values at a given time and place will obviously show some variation. Therefore, the stability of (15) and (17) when the aerosol parameters are allowed to vary over ranges which are still likely for ambient aerosol should be investigated.

1. *Variation in* D_0 (*aero*)

Figure 4 shows the variation in D_0(aero) when the six parameters representing density, mass median diameter, and geometrical standard deviation for both aerosol modes are separately allowed to vary. Figure 5 shows the corresponding result for D_0(aero) under variation of the refractive index. Although not shown, it was verified that setting the refractive index for the coarse mode different from that of the fine mode has virtually no effect on the value of D_0(aero). The insensitivity of the extinction coefficient to the refractive index of the coarse mode has been emphasized

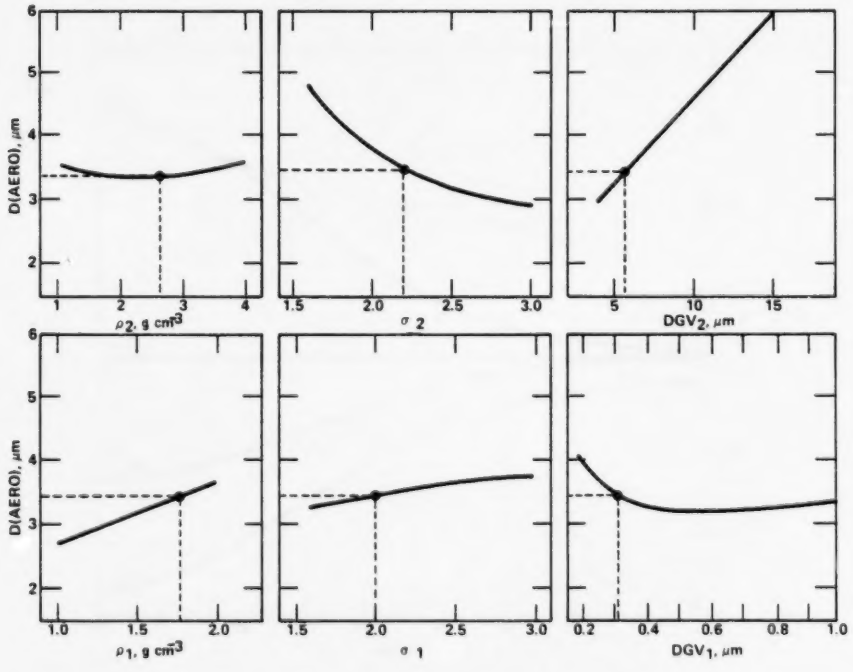

Fig. 4. Dependence of D_0 (aero) on the density, geometrical standard deviation, and mass median diameter of the coarse mode (top row) and accumulation mode (bottom row). The standard value for each parameter is indicated by a vertical dotted line. Note the use of suppressed zeros for all abscissas and ordinates.

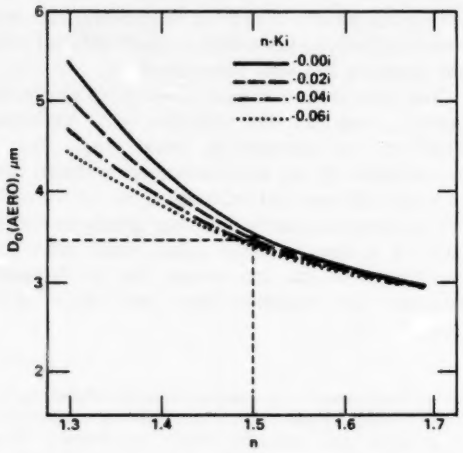

Fig. 5. Dependence of D_0 (aero) on the refractive index $m = n - ki$. The standard value 1.5–0.02i is indicated by the vertical dotted line. Note the use of suppressed zeros for both abscissa and ordinate.

previously by Willeke and Brockmann (1977).

Examination of Figs 4 and 5 shows that by and large the variation in D_0 (aero) falls in the range $3.44 \pm \frac{1}{0.5}$ μm. Variation in the parameters of the accumulation mode is seen to have little effect on D_0 (aero). The remarkably small dependence of D_0 (aero) on ρ_2 (coarse density) comes about through D_0 (geo) being nearly proportional to $1/\sqrt{\rho_2}$, which is cancelled when the conversion to D_0 (aero) is made. The most extreme value shown for D_0 (aero) is 5.86 μm, corresponding to a mass median diameter for the coarse mode of 15 μm. Because of the great difficulty in sampling coarse particles from the ambient atmosphere, any measurement of the mass median diameter is likely to be systematically low, which of course also applies to the value given in Table 1. For this reason the optimum value for D_0 (aero) should probably be somewhat larger than 3.44 μm.

2. *Variation in* M_f/b_{ep}

Equation (17) indicates that the ratio M_f/b_{ep} depends on 4 parameters of the fine aerosol mode—the

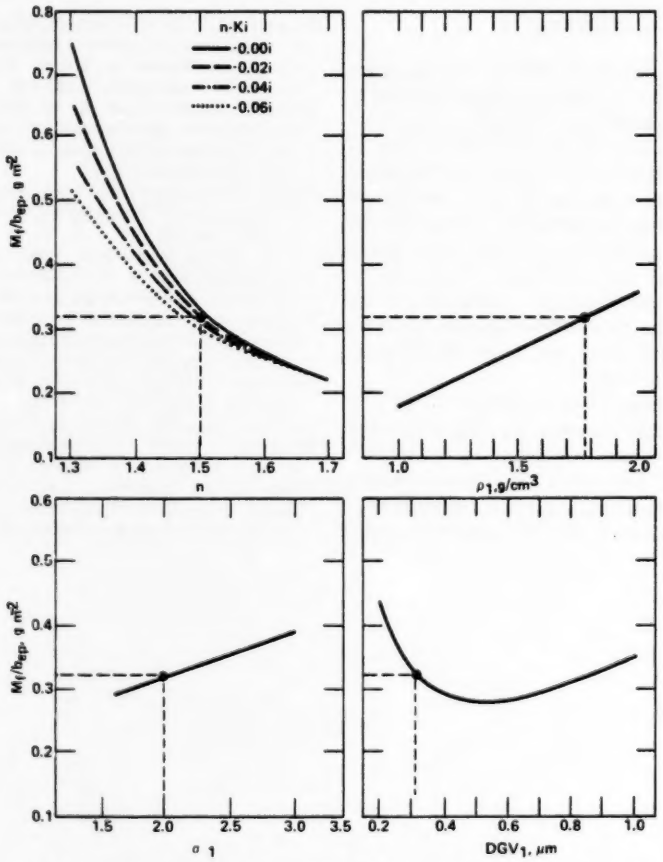

Fig. 6. Dependence of M_f/b_{ep} on the refractive index, density, geometrical standard deviation, and mass median diameter of the accumulation mode. The standard value for each parameter is indicated by a vertical dotted line. Note the use of suppressed zeros for all abscissas and ordinates.

density, mass median diameter, geometrical standard deviation and refractive index. The variation in the ratio corresponding to independent changes in each of the parameters is shown in Fig. 6. For the parameter ranges chosen changes in the mass median diameter and geometrical standard deviation cause the least change in M_f/b_{ep}. This conclusion for the mass median diameter has been noted previously by Waggoner and Weiss (1980). The variation in M_f/b_{ep} with density and refractive index is larger, amounting to a factor of 3 at the extremes of the latter parameter's variation. In actuality, typical ranges for the density and refractive index are probably smaller than those suggested in Fig. 6. Furthermore, the method of independent variation ignores the effect of correlations between parameters. For example, with a hygroscopic aerosol, increasing humidity will cause both ρ_1 and I_1 in (17) to decrease, with their ratio remaining relatively constant. Thus, in the extreme case of a pure water aerosol, M_f/b_{ep} has the value $0.32\,g\,m^{-2}$, the same as for the dry aerosol case of (18).

SUMMARY

The principal result of this study is that the upper particle size of the fine particle aerosol can be chosen so that the extinction coefficient is directly proportional to the fine mass concentration, and is independent of the coarse mass concentration whatever the magnitude of the latter. The result depends on the bimodal nature of the ambient aerosol size distribution. When the size parameters of the modes are set equal to their "grand average continental" values (Whitby, 1978) a value of $3.4\,\mu m$ (aerodynamic diameter) is obtained for the optimum fine-coarse cutpoint. Plausible changes in the values of the modal parameters affect the optimum cutpoint, but not strongly.

A secondary result is the evaluation of the proportionality constant between the extinction coefficient and the fine mass concentration, for which a value of $0.32\,g\,m^{-2}$ is obtained. The value is consistent with a current experimental value based on integrating nephelometer measurements. The sensitivity of the theoretical value to changes in the modal parameters, and the effect of a fine-coarse cutpoint different from the optimum one, were investigated,

The practical importance of the study's principal result is apparent. The inference of an extinction coefficient (or alternatively, visual range) from a measurement of fine mass concentration should have as much validity in arid undeveloped regions where the coarse aerosol concentrations may greatly exceed the fine, as it does in more urban areas where the importance of the fine aerosol and its dominant influence on extinction have been more easily recognizable.

Acknowledgements—I am pleased to acknowledge the important contributions to this article that have resulted from suggestions and criticisms made by Warren White (Washington University), Alan Waggoner (University of Washington), Tom Dzubay and John Bachmann (U.S. EPA). A conjecture by Robert Stevens (U.S. EPA) provided the impetus for this work.

REFERENCES

Brockmann J. E. (1976) Extinction coefficients of atmospheric aerosols and use in calculating visual range and solar beam attenuation. Master of Science thesis, University of Minnesota, Minneapolis, Minnesota.

Dzubay T. G. and Clubb K. W. (1980) Comparison of telephotometer measurements of extinction coefficients with scattering and absorption coefficients. *Atmospheric Environment* **15**, 2617–2624.

Frederick E. R., Editor (1979) View on visibility—regulatory and scientific. APCA Specialty Conference Proceedings. Denver, Colorado.

Lewis C. W. and Macias E. S. (1980) Composition of size-fractionated aerosol in Charleston, West Virginia. *Atmospheric Environment* **14**, 185–194.

Waggoner A. P. and Weiss R. E. (1980) Comparison of fine particle mass concentration and light scattering extinction in ambient aerosol. *Atmospheric Environment* **14**, 623–626.

Wedding J. B., Weigand M., John W. and Wall S. (1980) Sampling effectiveness of the inlet to the dichotomous sampler. *Envir. Sci. Technol.* **14**, 1367–1370.

Whitby K. T. (1978) The physical characteristics of sulfur aerosols. *Atmospheric Environment* **12**, 135–159.

Willeke K. and Brockmann J. E. (1977) Extinction coefficients for multimodal atmospheric particle size distributions. *Atmospheric Environment* **11**, 995–999.